

Ejercicios Método Simplex

Problem # 1

$$\text{Max } Z = 3x_1 + x_2 + 2x_3$$

s. t.

$$x_1 + x_2 + 2x_3 \leq 260$$

$$2x_2 + x_3 \leq 400$$

$$2x_1 + x_3 \leq 80$$

Sol.-

$$\text{Max } Z = 3x_1 + x_2 + 2x_3$$

s. t.

$$x_1 + x_2 + 2x_3 + h_1 = 260$$

$$2x_2 + x_3 + h_2 = 400$$

$$2x_1 + x_3 + h_3 = 80$$

	Z	x_1	x_2	x_3	h_1	h_2	h_3	RHS
		↓						
Z	1	-3	-1	-2	0	0	0	0
h_1	0	1	1	2	1	0	0	260
h_2	0	0	2	1	0	1	0	400
h_3	0	2	0	1	0	0	1	80
			↓					
Z	1	0	-1	-1/2	0	0	3/2	120
h_1	0	0	1	3/2	1	0	-1/2	220
h_2	0	0	2	1	0	1	0	400
x_1	0	1	0	1/2	0	0	1/2	40
Z	1	0	0	0	0	1/2	3/2	320
h_1	0	0	0	1	1	-1/2	-1/2	20
x_2	0	0	1	1/2	0	1/2	0	200
x_1	0	1	0	1/2	0	0	1/2	40

$$x_1 = 40$$

$$x_2 = 200$$

$$x_3 = 0$$

$$Z = 320$$

Problem # 2

$$\text{Max } Z = 2x_1 + 2x_2 + x_3$$

s. t.

$$2x_1 + x_2 + x_3 \leq 300$$

$$x_1 + x_3 \leq 500$$

$$x_1 + x_2 \leq 160$$

Sol.-

$$\text{Max } Z = 2x_1 + 2x_2 + x_3$$

s. t.

$$2x_1 + x_2 + x_3 + h_1 = 300$$

$$x_1 + x_3 + h_2 = 500$$

$$x_1 + x_2 + h_3 = 160$$

	Z	x_1	x_2	x_3	h_1	h_2	h_3	
			↓					
Z	1	-2	-2	-1	0	0	0	0
h_1	0	2	1	1	1	0	0	300
h_2	0	1	0	1	0	1	0	500
h_3	0	1	1	0	0	0	1	160
				↓				
Z	1	0	0	-1	0	0	2	320
h_1	0	1	0	1	1	0	-1	140
h_2	0	1	0	1	0	1	0	500
x_2	0	1	1	0	0	0	1	160
Z	1	1	0	0	1	0	1	460
x_3	0	1	0	1	1	0	-1	140
h_2	0	0	0	0	-1	1	1	360
x_2	0	1	1	0	0	0	1	160

$$x_1 = 0$$

$$x_2 = 160$$

$$x_3 = 140$$

$$Z = 460$$

Problem # 3

$$\text{Max } Z = 3x_1 + 4x_2 + 2x_3$$

s. t.

$$x_1 + x_2 + 2x_3 \leq 120$$

$$2x_1 + x_2 \leq 80$$

$$x_1 + 2x_2 + 2x_3 \leq 300$$

$$x_2 + x_3 \leq 150$$

Sol.-

$$\text{Max } Z = 3x_1 + 4x_2 + 2x_3$$

s. t.

$$x_1 + x_2 + 2x_3 + h_1 = 120$$

$$2x_1 + x_2 + h_2 = 80$$

$$x_1 + 2x_2 + 2x_3 + h_3 = 300$$

$$x_2 + x_3 + h_4 = 150$$

	Z	x_1	x_2	x_3	h_1	h_2	h_3	h_4	
			↓						
Z	1	-3	-4	-2	0	0	0	0	0
h_1	0	1	1	2	1	0	0	0	120
h_2	0	2	1	0	0	1	0	0	80
h_3	0	1	2	2	0	0	1	0	300
h_4	0	0	1	1	0	0	0	1	150
				↓					
Z	1	5	0	-2	0	4	0	0	320
h_1	0	-1	0	2	1	-1	0	0	40
x_2	0	2	1	0	0	1	0	0	80
h_3	0	-3	0	2	0	-2	1	0	140
h_4	0	-2	0	1	0	-1	0	1	70
Z	1	4	0	0	1	3	0	0	360
x_3	0	-1/2	0	1	1/2	-1/2	0	0	20
x_2	0	2	1	0	0	1	0	0	80
h_3	0	-2	0	0	-1	-1	1	0	100
h_4	0	-3/2	0	0	-1/2	-1/2	0	1	50

$$x_1 = 0$$

$$x_2 = 80$$

$$x_3 = 20$$

$$x_4 = 0$$

$$Z = 360$$

Problem # 4

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3$$

s. t.

$$2x_1 + x_2 + x_3 \leq 160$$

$$2x_1 + 3x_3 \leq 90$$

$$x_1 + 2x_2 + x_3 = 50$$

Sol.-

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3 - MA_3$$

s. t.

$$2x_1 + x_2 + x_3 + h_1 = 160$$

$$2x_1 + 3x_3 + h_2 = 90$$

$$x_1 + 2x_2 + x_3 + A_3 = 50$$

	Z	x_1	x_2	x_3	h_1	h_2	A_3	
			↓					
Z	1	$-M-3$	$-2M-5$	$-M-4$	0	0	0	$-50M$
h_1	0	2	1	1	1	0	0	160
h_2	0	2	0	3	0	1	0	90
A_1	0	1	2	1	0	0	1	50
				↓				
Z	1	$-1/2$	0	$-3/2$	0	0	$M+5/2$	125
h_1	0	$3/2$	0	$1/2$	1	0	$-1/2$	135
h_2	0	2	0	3	0	1	0	90
x_2	0	$1/2$	1	$1/2$	0	0	$1/2$	25
Z	1	$1/2$	0	0	0	$1/2$	$M+5/2$	170
h_1	0	$7/6$	0	0	1	$-1/6$	$-1/2$	120
x_3	0	$2/3$	0	1	0	$1/3$	0	30
x_2	0	$1/6$	1	0	0	$-1/6$	$1/2$	10

$$x_1 = 0$$

$$x_2 = 10$$

$$x_3 = 30$$

$$Z = 170$$

Problem # 5

$$\text{Max } Z = x_1 + 2x_2 + x_3 + 3x_4$$

s. t.

$$2x_1 + x_2 + x_3 \leq 300$$

$$x_2 + 2x_3 + 2x_4 \leq 400$$

$$x_1 + x_4 = 120$$

Sol.-

$$\text{Max } Z = x_1 + 2x_2 + x_3 + 3x_4 - MA_3$$

s. t.

$$2x_1 + x_2 + x_3 + h_1 = 300$$

$$x_2 + 2x_3 + 2x_4 + h_2 = 400$$

$$x_1 + x_4 + A_3 = 120$$

	Z	x_1	x_2	x_3	x_4	h_1	h_2	A_3	
					↓				
Z	1	-M-1	-2	-1	-M-3	0	0	0	-120M
h_1	0	2	1	1	0	1	0	0	300
h_2	0	0	1	2	2	0	1	0	400
A_3	0	1	0	0	1	0	0	1	120
			↓						
Z	1	2	-2	-1	0	0	0	M+3	360
h_1	0	2	1	1	0	1	0	0	300
h_2	0	-2	1	2	0	0	1	-2	160
x_4	0	1	0	0	1	0	0	1	120
		↓							
Z	1	-2	0	3	0	0	2	M+1	680
h_1	0	4	0	-1	0	1	-1	2	140
x_2	0	-2	1	2	0	0	1	-2	160
x_4	0	1	0	0	1	0	0	1	120
Z	1	0	0	5/2	0	1/2	3/2	M	750
x_1	0	1	0	-1/4	0	1/4	-1/4	1/2	35
x_2	0	0	1	3/2	0	1/2	1/2	-1	230
x_4	0	0	0	1/4	1	-1/4	1/4	1/2	85

$$x_1 = 35$$

$$x_2 = 230$$

$$x_3 = 0$$

$$x_4 = 85$$

$$Z = 750$$

Problem # 6

$$\text{Max } Z = 2x_1 + 3x_2 + 4x_3$$

s. t.

$$x_1 + 2x_2 + x_3 \leq 300$$

$$2x_1 + x_2 + x_3 \leq 400$$

$$x_1 + x_3 \geq 120$$

$$x_2 + x_3 = 80$$

Sol.-

$$\text{Max } Z = 2x_1 + 3x_2 + 4x_3$$

$$x_1 + 2x_2 + x_3 + h_1 = 300$$

$$2x_1 + x_2 + x_3 + h_2 = 400$$

$$x_1 + x_3 - h_3 + A_3 = 120$$

$$x_2 + x_3 + A_4 = 80$$

	Z	x_1	x_2	x_3	h_1	h_2	h_3	A_3	A_4	
				↓						
Z	1	-M-2	-M-3	-2M-4	0	0	M	0	0	-200 M
h_1	0	1	2	1	1	0	0	0	0	300
h_2	0	2	1	1	0	1	0	0	0	400
A_3	0	1	0	1	0	0	-1	1	0	120
A_4	0	0	1	1	0	0	0	0	1	80
		↓								
Z	1	-M-2	-M-1	0	0	0	M	0	2M+4	320-40 M
h_1	0	1	1	0	1	0	0	0	-1	220
h_2	0	2	0	0	0	1	0	0	-1	320
A_3	0	1	-1	0	0	0	-1	1	-1	40
x_3	0	0	1	1	0	0	0	0	1	80
							↓			
Z	1	0	-1	0	0	0	-2	M+2	M+2	400
h_1	0	0	2	0	1	0	1	-1	0	180
h_2	0	0	2	0	0	1	2	-2	1	240
x_1	0	1	-1	0	0	0	-1	1	-1	40
x_3	0	0	1	1	0	0	0	0	1	80
Z	1	0	1	0	0	1	0	M	M+3	640
h_1	0	0	1	0	1	-1/2	0	0	-1/2	60
h_3	0	0	1	0	0	1/2	1	-1	1/2	120
x_1	0	1	0	0	0	1/2	0	0	-1/2	160
x_3	0	0	1	1	0	0	0	0	1	80

$$x_1 = 160$$

$$x_2 = 0$$

$$x_3 = 80$$

$$Z = 640$$

Problem # 7

Max $Z = 2x_1 + x_2 + 3x_3 + 2x_4$

s. t.

$2x_1 + 2x_2 + x_3 + x_4 \leq 400$

$x_1 + x_2 + x_4 \geq 120$

$2x_2 + x_3 + x_4 \geq 160$

Sol.-

Max $Z = 2x_1 + x_2 + 3x_3 + 2x_4 - MA_2 - MA_3$

s. t.

$2x_1 + 2x_2 + x_3 + x_4 + h_1 = 400$

$x_1 + x_2 + x_4 - h_2 + A_2 = 120$

$2x_2 + x_3 + x_4 - h_3 + A_3 = 160$

	Z	x_1	x_2	x_3	x_4	h_1	h_2	h_3	A_2	A_3	
			↓								
Z	1	-M-2	-3M-1	-M-3	-2M-2	0	M	M	0	0	-280M
h_1	0	2	2	1	1	1	0	0	0	0	400
A_2	0	1	1	0	1	0	-1	0	1	0	120
A_3	0	0	2	1	1	0	0	-1	0	1	160
		↓									
Z	1	-M-2	0	$1/2M - 5/2$	$-1/2M - 3/2$	0	M	$-1/2M - 1/2$	0	$3/2M + 1/2$	80-40M
h_1	0	2	0	0	0	1	0	1	0	-1	240
A_2	0	1	0	-1/2	1/2	0	-1	1/2	1	-1/2	40
x_2	0	0	1	1/2	1/2	0	0	-1/2	0	1/2	80
			↓								
Z	1	0	0	-7/2	-1/2	0	-2	1/2	M+2	M-1/2	160
h_1	0	0	0	1	-1	1	2	0	-2	0	160
x_1	0	1	0	-1/2	1/2	0	-1	1/2	1	-1/2	40
x_2	0	0	1	1/2	1/2	0	0	-1/2	0	1/2	80
			↓								
Z	1	0	0	0	-4	7/2	5	1/2	M-5	M-1/2	720
x_3	0	0	0	1	-1	1	2	0	-2	0	160
x_1	0	1	0	0	0	1/2	0	1/2	0	-1/2	120
x_2	0	0	1	0	1	-1/2	-1	-1/2	1	1/2	0
			↓								
Z	1	0	4	0	0	3/2	1	-3/2	M-1	M-3/2	720
x_3	0	0	1	1	0	1/2	1	-1/2	-1	1/2	160
x_1	0	1	0	0	0	1/2	0	1/2	0	-1/2	120
x_4	0	0	1	0	1	-1/2	-1	-1/2	1	1/2	0
			↓								
Z	1	3	4	0	0	3	1	0	M-1	M	1080
x_3	0	1	1	1	0	1	1	0	-1	0	280
h_3	0	2	0	0	0	1	0	1	0	-1	240
x_4	0	1	1	0	1	0	-1	0	1	0	120

Problem # 8

Maximizar $Z = 2x_1 + 3x_2 + 4x_3$

s. t.

$x_1 + 2x_2 - x_3 \leq 300$

$2x_1 + x_2 + x_3 \geq 400$

$x_1 + x_3 \geq 120$

Sol.-

Max $Z = 2x_1 + 3x_2 + 4x_3 - MA_2 - MA_3$

s. t.

$x_1 + 2x_2 - x_3 + h_1 = 300$

$2x_1 + x_2 + x_3 - h_2 + A_2 = 400$

$x_1 + x_3 - h_3 + A_3 = 120$

	Z	x_1	x_2	x_3	h_1	h_2	h_3	A_2	A_3	
		↓								
Z	1	$-3M-2$	$-M-3$	$-2M-4$	0	M	M	0	0	$-520M$
h_1	0	1	2	-1	1	0	0	0	0	300
A_2	0	2	1	1	0	-1	0	1	0	400
A_3	0	1	0	1	0	0	-1	0	1	120
							↓			
Z	1	0	$-M-3$	$M-2$	0	M	$-2M-2$	0	$3M+2$	$240-160M$
h_1	0	0	2	-2	1	0	1	0	-1	180
A_2	0	0	1	-1	0	-1	2	1	-2	160
x_1	0	1	0	1	0	0	-1	0	1	120
				↓						
Z	1	0	-2	-3	0	-1	0	$M+1$	M	400
h_1	0	0	$3/2$	$-3/2$	1	$1/2$	0	$-1/2$	0	100
h_3	0	0	$1/2$	$-1/2$	0	$-1/2$	1	$1/2$	-1	80
x_1	0	1	$1/2$	$1/2$	0	$-1/2$	0	$1/2$	0	200
Z	1	6	1	0	0	4	0	$M+4$	M	1600
h_1	0	3	3	0	1	-1	0	1	0	700
h_3	0	1	1	0	0	-1	1	1	-1	280
x_3	0	2	1	1	0	-1	0	1	0	400
		-6	-1	0	0	4	0	-M-4	-M	Z - 1600
						↑				

Solución Infinita, ya que h_2 puede crecer indefinidamente